HYDRAULIC DRAG IN A STATIONARY BED OF ION-EXCHANGE RESIN

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On the basis of the drag law for a bed of granular material and from a relation established for the bed porosity as a function of the pressure drop, an empirical formula is derived which yields the most accurate value for the hydraulic drag in a stationary bed of an ion-exchange resin, with its deformation under pressure taken into account.

The hydraulic drag of a granular bed is usually calculated according to the formula [1]

$$\frac{\Delta p}{l} = \lambda \frac{a_0}{\varepsilon_0^3} \cdot \frac{u^2}{2g} \gamma, \tag{1}$$

where the drag coefficient is related to the Reynolds number as follows:

$$\lambda = \frac{k_v}{\mathrm{Re_e}} + k_i \tag{2}$$

The applicability of this formula with the coefficient in it to a bed with an ion-exchange resin is not quite obvious. The resin grains, with many minute pores, swell in water in a sponge-like manner. This determines the specific characteristics of the grain surface as well as deformability of grains under the pressure of filtrating liquid. Experience in the use of ionite filters has shown that a resin bed is quite compressible during filtration. The bed characteristics (porosity ε and specific surface a) depend then on the conditions under which the filtrating stream acts on the bed grains. A theoretical analysis of the flow of liquid through a porous medium [2, 3] has shown that the action here is due to the filtrational force, which per unit bed volume with an ionite filter (uniform and steady filtration under a pressure head) is

$$f = \gamma I. \tag{3}$$

Inasmuch as the bed grains support one another to stay in position, the filtration forces are transmitted from grain to grain and add up to a net force in the direction of flow. Thus, the stress due to filtration forces at any bed section normal to the filtration flow is characterized by a loss of pressure Δp or of momentum in the stream down to that section. The total compressive stress σ in a bed represents the

TABLE 1. Specimen Characteristics

Specimen number	Grade of resin or copolymer	Equivalent grain diameter $d_e \cdot 10^3$ m	Nonuniformity factor, K_{nu}	Bed porosity, ε_0	Specimen number	Grade of resin or copolymer	Equivalent grain diameter $d_e \cdot 10^3$ m	Nonuniformity factor, K_{nu}	Bed porosity, ε_0
1	KU-2x8	0,354	1,17	0,388	9	AV-17x8	0,516	1,52	0,380
2		0,500	1,33	0,390	10	KhMS-8p	0,500	1,33	0,388
3		0,544	1,81	0,370	11	AV-17x12p	0,790	1,40	0,385
4		0,670	1,56	0,375	12	AVKh-12p	0,790	1,40	0,380
5		0,788	2,17	0,365	13	AVF-12p	0,790	1,40	0,383
6		0,730	1,21	0,395	14	AN-22x12p	0,790	1,40	0,386
7		0,770	1,53	0,380	15	KhMS-12p	0,790	1,40	0,378
8		0,920	1,12	0,400	16	KhMS-24p	0,790	1,40	0,375

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TABLE 2. Test Data Pertaining to the Hydraulic Drag Coefficient and the Bed Compressibility*

Specimen No.	k_v	k_i	m	$n\cdot10^2$, m^2/MN	Specimen No.	k_v	k_i	m	$n\cdot10^2$, m^2/MN
1	39,0	0,750			9	38,0	1,00	0,968	—32,6
2	37,5	0,895			10	—	—	0,965	—28,5
3	39,9	0,905	0,965–0,974	—29,6——30,6	11	39,0	0,675	0,971	—27,5
4	39,0	0,873			12	42,5	0,705	0,975	—26,5
5	41,5	0,950			13	40,0	0,628	0,987	—24,5
6	41,0	0,955			14	39,0	0,660	0,970	—25,5
7	38,0	0,985			15	40,0	0,792	0,962	—26,5
8	37,9	1,00			16	41,0	0,565	0,985	—17,3

*Specimen numbers here correspond to those in Table 1.

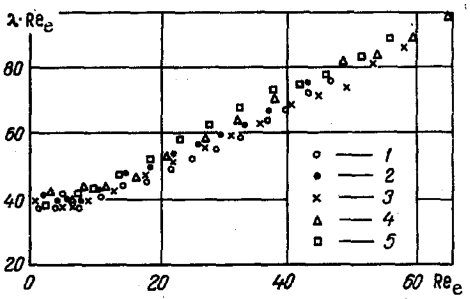

Fig. 1. Relation $\lambda \cdot Re_e = f(Re_e)$: 1) specimen No. 2; 2) specimen No. 3; 3) specimen No. 4; 4) specimen No. 5; 5) specimen No. 6.

sum of filtrational and gravitational stresses per unit bed surface area, with the Archimedes force discounted:

$$\sigma = \Delta p + (\rho_g - \rho)(1 - \varepsilon_0)\, lg. \tag{4}$$

The second component here is rather insignificant relative to the first one. For filtration of water through a grade KU-2×8 ionite bed 1 m high at a velocity of 11-14 mm/sec, data from practical experience yield $\Delta p = 0.10\text{-}0.12$ MN/m^2 and the weight of a bed 0.0025 MN/m^2, with the Archimedes force discounted and with the density of grains assumed $\rho_g = 1400$ kg/m^3 (actually, $\rho_g < 1400$ kg/m^3) at a porosity $\varepsilon = 0.35$, amounts to 2.1-2.5% of Δp. At filtration velocities higher than that, the weight of an ionite bed with the Archimedes force included constitutes an even smaller fraction of Δp. For further simplification, we will assume that $\sigma = \Delta p$ and that the porosity as well as the specific surface are functions of Δp only. In the case of a high bed of resin with a large loss of pressure, formula (1) is applicable in differential form only:

$$\frac{d(\Delta p)}{dl} = \lambda \frac{a}{\varepsilon^3} \cdot \frac{u^2}{2g} \gamma.$$

In order to integrate Eq. (1a), one must know how the porosity and the specific surface depend on the pressure drop and how the hydraulic drag depends on the equivalent Reynolds number in a resin bed. In earlier published studies concerning the hydraulic drag in resin beds [4-7], the filtration velocities were moderate and compression of the bed was not taken into account. Here we will present the results of studies pertaining to those relations and to drag calculations according to Eq. (1a) for resin beds.

For the study we selected specimens of various resin grades with spherical grains (grains of irregular shape were rejected). For comparison, we also examined specimens of macroporous chloromethylated copolymers (also with spherical grains) containing 8, 12, or 24% divinyl benzene (DVB) and designated respectively as KhMS-8p, KhMS-12p, and KhMS-24p.

The characteristics of these test specimens are listed in Table 1. The equivalent diameter d_e of the resin grains and their nonuniformity factor K_{nu} were established by the methods in [8] and [9]. The bed porosity ε_0 was calculated according to the formula

$$\varepsilon_0 = 1 - \frac{\rho_{bo}}{\rho_g}. \tag{5}$$

Hydraulic Drag Coefficient. The value of λ at various values of the Reynolds number Re_e was determined from the results of pressure drop and velocity measurements in water filtrating through a resin bed 100 mm high and 30 mm in diameter. With a diameter of this size, according to calculations checked against the formula in [10], the wall effect could be disregarded. The moderate bed height and the correspondingly moderate pressure drops along the flow made it permissible to disregard the bed compression, to regard the specific surface as equal to the sum of the surfaces of all grains per unit bed volume (point contacts between grains), and to assume that $\varepsilon = \varepsilon_0$. We have, therefore, used formula (1) for calculating λ. The filtration velocity was varied from 2.78 to 72.3 mm/sec. The value of λ was taken as the average of five measurements in each operating mode. Moreover, a test specimen was loaded each time from the

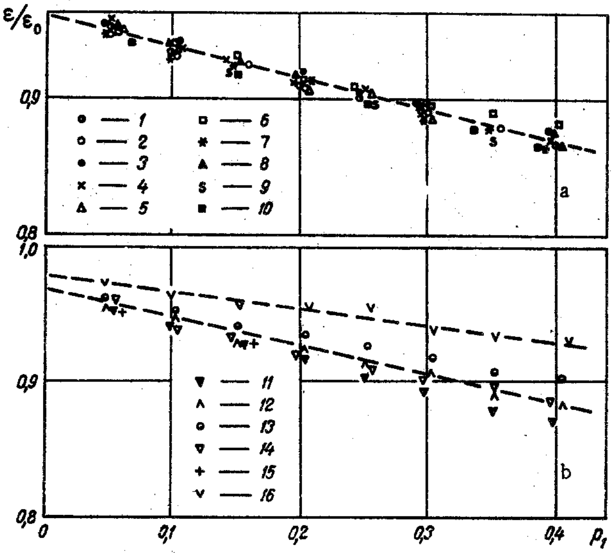

Fig. 2. Relative porosity $\varepsilon/\varepsilon_0$ as a function of the load p_1 (MN/m^2): (a) for specimens containing 8% DVB, (b) for specimens containing 12 or 24% DVB (the numbers correspond to respective specimens 1-16).

start. For a convenient determination of k_v and k_i, we have plotted graphs of $\lambda \cdot Re_e = f(Re_e)$. These graphs are shown in Fig. 1 for a few specimens. In all cases, also for many other granular materials, at low values of Re_e (horizontal range in Fig. 1) the relation between λ and Re_e is

$$\lambda = \frac{k_v}{Re_e}, \tag{6}$$

corresponding to a linear drag law. The trend of this relation changes at $Re_e = Re_{cr} = 7.0\text{–}7.5$, above which inertia forces enter into play so that the relation between λ and Re_e becomes (Fig. 1)

$$\lambda = \frac{k_v}{Re_e} + k_i\left(\frac{Re_e - Re_{cr}}{Re_e}\right). \tag{7}$$

The values of k_v and k_i for our test specimens lie within 37.5-42.5 and 0.565-1.000 (Table 2) respectively. For a bed of plain balls, a generalization of many test data [1] has yielded $k_v = 8 \cdot 4.2\text{–}8 \cdot 4.8$ and $k_i = 0.39\text{–}0.61$ respectively.

A comparison shows that, for a resin bed during filtration under the predominance of viscous forces, the values of k_v come close to those for plain balls. The value of k_i, which characterizes a flow under the predominance of inertia forces and most strongly affected by the surface condition of grains, is much higher for resin beds.

It is to be noted that the conventional formula (2) for the flow of liquid under the predominance of inertia forces does most accurately correspond to test conditions at a Reynolds number Re_e much higher than Re_{cr}. For ion-exchange beds, of practical interest is mainly filtration with Re_e up to 25-30 ($u \approx 30 \cdot 10^{-3}$ mm/sec) and, therefore, relation (7) is in this case more appropriate.

Bed Porosity ε. A resin bed in a filter is bounded between the apparatus walls and, therefore, deformed only in the direction of flow. In view of this, any volume element of bed material may be seen as existing in a closed space under a compressive force equal to the filtration force.

In order to study how the bed porosity depends on the compressive load, we used a so-called compression cell consisting of a metallic cylinder 50 mm in diameter with a perforated plunger. Into this cylinder (underneath the plunger) was poured resin cast in water. A compressive load was then applied to the plunger, causing the bed to deform. Excess water oozed freely out through the holes in the plunger. The density of grains was assumed constant and the bed deformation dl to be only due to a decrease in porosity, so that ε could be calculated as

$$\varepsilon = 1 - \frac{\rho_{bo}}{\rho_g} \cdot \frac{1}{1 - \frac{\Delta l}{l}}. \tag{8}$$

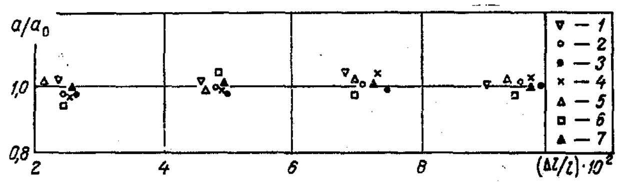

Fig. 3. Relative specific surface of a/a_0 of a compressed bed (points 1-6 correspond to specimens number 1-6 respectively, point 7 corresponds to specimen number 8).

The maximum load on the bed was 0.49 MN/m^2. For convenience, the data were subsequently evaluated and generalized in terms of the relative porosity $\varepsilon/\varepsilon_0$. The graphs of log $\varepsilon/\varepsilon_0$ versus pressure p_1 on the bed are straight lines for all specimens (Fig. 2a, b). These graphs can be represented by the equation

$$\varepsilon/\varepsilon_0 = m \cdot \exp(n\Delta p). \tag{9}$$

The values of m and n are shown in Table 2. Comparing the values for various specimens shows that the compressibility of a bed (slope of the lines, i.e., n) is essentially a function of the DVB content in the ionite. Thus, the mean value of n is 0.306 m^2/MN for 8% DVB, 0.265 m^2/MN for 12% DVB, and 0.173 m^2 /MN for 24% DVB (dashed lines in Fig. 2). This kind of relation between the porosity of a compressed bed and the DVB content in it is indicative of deformation at the grains directly. The deformation of a bed with a random ("normal" [1]) packing of grains is partly due to the redistribution of grains. This explanations the greater change in porosity under light loads, and some departure of $\varepsilon/\varepsilon_0$ from unity according to Eq. (9) at $p_1 = 0$ $(m < 1)$.

When applied to a filter bed where the filtration force is causing compression, i.e., where $p_1 = \Delta p$, Eq. (9) becomes

$$\varepsilon/\varepsilon_0 = m \cdot \exp(np_1). \tag{10}$$

<u>Specific Surface *a*.</u> During the deformation of a bed, the contact points between grains become contact areas whose direct determination is quite difficult. This, accordingly, causes difficulties in the determination of the specific contact surface between phases in a compression bed. In our study we applied an indirect method of determining a: from the flow parameters of laminar filtration through a bed under artificially maintained uniform compression.

In the light of these arguments, relation (1) seems applicable to filtration through a high bed. For a liquid flowing under the predominance of viscous forces, when $\lambda = k_V/Re_e$, relation (1) becomes

$$\frac{\Delta p}{l} = \frac{k_V}{8} \cdot \frac{a_0^2}{\varepsilon_0^3} \mu u.$$

It follows from here that

$$a/a_0 = \sqrt{\frac{\Delta p/l}{(\Delta p/l)_0} \left(\frac{\varepsilon}{\varepsilon_0}\right)^3 \frac{\mu_0}{\mu} \cdot \frac{u_0}{u}}, \tag{11}$$

where the symbols with the subscript "o" refer to filtration through an uncompressed bed.

For filtration through a compressed bed, the hydraulic drag coefficient was measured in a columm. Here compression was produced by means of a perforated plunger, inserted into the column above the resin bed and pushed down along a threaded bushing. The plunger travel during compression was measured with a gage on the basis of the thread pitches traversed. The bed porosity ε was then calculated according to Eq. (8). The relative compression $\Delta l/l$ was varied from 0 to 0.1.

The tests were performed with specimens of grade KU-2×8 resin of various fractional compositions. The test results are shown in Fig. 3, indicating that a is almost equal to a_0 regardless of the fractional composition and of the compression ratio (within the test range). An explanation for this could be that the decrease in a during an increase in the contact surface between grains is compensated by an increase in a due to the larger number of grains per unit bed volume. Assuming, on the basis of the test data, that a remains constant regardless of the compression ratio, we express this parameter in terms of porosity ε_0 and the specific grain surface

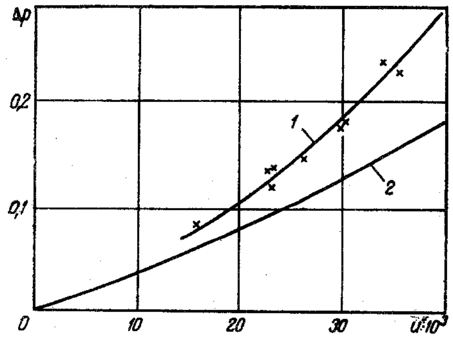

Fig. 4. Comparison of calculated and experimental data on filtration through layer KU 2 × 8 in pilot filter: 1) calculated curve by (14); 2) calculated curve by (1); points, experimental data.

$$a = \frac{6(1-\varepsilon_0)}{d_e}. \tag{12}$$

Calculation of the Hydraulic Drag in a High Bed. With (7), (10), and (12) taken into account, Eq. (1a) becomes

$$\frac{d(\Delta p)}{dl} = \frac{1}{\varepsilon_0^3 m^3 \exp(3n\Delta p)}\left[\frac{4{,}5(k_v - k_i \mathrm{Re}_{cr})(1-\varepsilon_0)^2\mu}{d_g^2}u + \frac{3k_i(1-\varepsilon_0)\rho}{d_g}u^2\right]. \tag{13}$$

After separation of variables and integration over the bed height from 0 to l in the direction of flow corresponding to a change in drag from 0 to Δp, we obtain the following final expression for the drag in a bed where $\mathrm{Re}_e > \mathrm{Re}_{cr}$:

$$\Delta p = \frac{1}{3n}\ln\left[\left(\frac{4{,}5(k_v - k_i \mathrm{Re}_{cr})(1-\varepsilon_0)^2\mu}{d_g^2}u + \frac{3k_i(1-\varepsilon_0)\rho}{d_g}u^2\right)\frac{3nl}{m^3\varepsilon_0^3} + 1\right]. \tag{14}$$

Obviously, with the inertial term missing ($\mathrm{Re}_e > \mathrm{Re}_{cr}$), the drag formula simplifies appreciably.

The drag in a bed 0.8 m high has been plotted in Fig. 4 (curve 1) for grade KU-2×8 ionite (d_e = 0.578 mm, K_{nu} = 1.9, and ε_0 = 0.352), according to Eq. (14), as a function of the filtration velocity. For the test values of k_v, k_i, m, and n were taken here the averages for five specimens of this particular grade resin (k_v = 39.2, k_i = 0.914, m = 0.97, and n = −0.3015 m^2/MN). The points on the diagram represent the test values for filtration of water through that grade ionite in a semiindustrial filter 0.6 mm in diameter. Curve 2 has been calculated according to Eq. (1) for plain granular materials. The values for k_v and k_i were in this case based on recommendations by M. É. Aerov [1]. The maximum deviation of test data from the calculated curves 1 and 2 was 8% and 36% respectively. Evidently, accounting for the bed deformation yields more accurate values for the hydraulic drag in a stationary bed of ion-exchange resin.

NOTATION

Δp	is the hydraulic drag in a bed (pressure drop along the flow);
l	is the bed height;
σ	is the compressive stress in a bed;
u	is the filtration velocity;
λ	is the hydraulic drag coefficient;
$\mathrm{Re}_e = 4u\rho/a\mu$	is the equivalent Reynolds number;
$k_v = 8K$	is the viscous component of the drag coefficient;
K	is the Koseni—Karman constant;
k_i	is the inertia component of the drag coefficient;
K_{nu}	is the nonuniformity factor referred to grain size distribution;
a	is the specific surface of a deformed bed;
a_0	is the specific surface of an undeformed bed;
ε	is the porosity of a deformed bed;
ε_0	is the porosity of an undeformed bed;
ρ_b	is the density of a deformed bed;
ρ_{b0}	is the density of an undeformed bed;
ρ_g	is the true density of resin grains;
d_e	is the equivalent diameter of grains in the bed;
γ	is the specific gravity of the filtrating liquid;

ρ is the density of the filtrating liquid;
μ is the dynamic viscosity of the filtrating liquid;
g is the acceleration due to gravity (9.81 m/sec^2).

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